R13

Code No: 113AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, September/October - 2023
MATHEMATICS - II
(Common to CE, AE, PTM)

Time: 3 hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Verify whether $\bar{f} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$ is irrotational vector or not? [2]

b) Determine $\nabla \bullet \left(r^n \bar{r}\right)$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $r = |\bar{r}|$. [3]

c) Which of the following functions is/are periodic? What is the period? [2]
$f(x) = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$, and $f(x) = \cos\left(2x\right) + 3$.

d) Find the inverse finite Fourier sine transform $f(x)$ if [3]
$F_s(n) = \frac{2\pi\left(-1\right)^{n-1}}{n^2}$, $n = 1, 2, ...,$ when $0 < x < \pi$

e) Define the method of least squares. [2]

f) Prove or disprove: $\left(E^{1/2} + E^{-1/2}\right)\left(1+\Delta\right)^{1/2} = 2 + \Delta$ with the usual notations. [3]

g) What is meant by transcendental equations? [2]

h) Using Newton-Raphson method, derive an iterative scheme to compute $\frac{1}{\mathrm{N}}$, where N is positive integer. [3]

i) Write the formula for the Runge-Kutta method of 4th order to find $y(x_0 + h)$ from the initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. [2]

j) Compute the integral $\int_0^6 \frac{1}{1+x^2}\, dx$ using two-point Gauss quadrature formulae. [3]

PART-B

(50 Marks)

2.a) Show that $\operatorname{div}\left(\operatorname{grad}\{r^m\}\right) = m(m+1)\, r^{m-2}$,
where m is a constant, $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $r = |\bar{r}|$.

b) Find the directional derivative of the function $\phi = x y^2 + y z^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \ln z - y^2 = -4$ at $(-1, 2, 1)$. [5+5]

OR

3.a) State Stokes theorem.

b) Verify the Stoke's theorem for the vector field $\vec{F} = x\,\vec{i} + z^2\,\vec{j} + y^2\,\vec{k}$ over the plane surface $x+y+z=1$ lying in the first octant. [2+8]

4.a) State Dirichlet conditions.

b) Obtain the half-range Fourier cosine and sine series for

$$f(x) = \begin{cases} x, & 0 < x < \dfrac{\pi}{2} \\ \pi - x, & \dfrac{\pi}{2} < x < \pi \end{cases}.$$ [5+5]

OR

5.a) State complex form of Fourier integral theorem.

b) Determine the Fourier sine transform of $f(x) = \dfrac{1}{x(a^2+x^2)}$. [2+8]

6. From the data given below, obtain the number of students whose weight is between 40 –50 and 90-100 using suitable interpolation formulae. [10]

Weights in kgs	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

OR

7. Using the given table below, determine the constants a and b by the method of least squares such that $y = a\,e^{bx}$. [10]

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

8. For the equation $x^3 - 4x + 1 = 0$,
a) Indicate all possible intervals in which the real roots exist?
b) Find out any one of the approximate roots correct up to three decimal places by using method of false position. [3+7]

OR

9.a) What is diagonally dominant property? State it.

b) Using the L-U decomposition method, solve the system of equations:
$x + 2y + 6z = 5;\ 2x + 6y + 4z = 13;\ 8x + y + 4z = 13.$ [2+8]

10. Use $h = 0.1$ to evaluate the value of y when $x = 1.3$ for $x^2 \dfrac{dy}{dx} + xy = 1,\ y(1) = 1$, using Taylor series method. Also, compare the numerical results with the exact values. [10]

OR

11. Solve the boundary value problem defined by $\dfrac{d^2y}{dx^2} - 64y + 10 = 0,\ y(0) = 0,\ y(1) = 0$ using the shooting method. [10]

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